Dr. Kamlesh Kumar (1) Date 29/04/21

B. Sc (Hons) Part - II, Paper - IV

Differential Equation of 1st order and 1st degree

A differential equation of the First order and First degree can be present as

$$Mdx + Ndy = 0$$

Where M and N are functions of x and y or arbitrary constants not involving the differential coefficients.

There are different method to solving the differential equation of the 1st order and 1st degree. we consider one of them -

Variables Separation Method

If the given eqn. $Mdx + Ndy = 0$ where M and N are functions of x and y or arbitrary constants but not of derivatives can be put in the form $f(x)dx + \phi(y)dy = 0$ where $f(x)$ is a function of x only and $\phi(y)$ is a function of y only, then we say that the variables are seperable. The solution of such equations can be easily found by direct integration, such that its solution is $\int f(x)dx + \int \phi(y)dy = K$, where K is an arbitrary constant.

ex (1) Solve $\frac{dy}{dx} + \sqrt{\frac{1-y^2}{1-x^2}} = 0$

Soln. Given, $\frac{dy}{dx} + \sqrt{\frac{1-y^2}{1-x^2}} = 0 \Rightarrow \frac{dy}{dx} = -\sqrt{\frac{1-y^2}{1-x^2}}$

$\Rightarrow \frac{dy}{\sqrt{1-y^2}} = -\frac{dx}{\sqrt{1-x^2}} \Rightarrow \int \frac{dy}{\sqrt{1-y^2}} = -\int \frac{dx}{\sqrt{1-x^2}}$

$\Rightarrow \sin^{-1}y = -\sin^{-1}x + c \Rightarrow \sin^{-1}x + \sin^{-1}y = c$ Ans.

Where c is constant

Ex (2) Solve $y\,dx - x\,dy = xy\,dx$ [Very Important For Exam.]

Soln. Given $y\,dx - x\,dy = xy\,dx$

$\Rightarrow x\,dy = y\,dx - xy\,dx \Rightarrow x\,dy = y(dx - x\,dx)$

$\Rightarrow x\,dy = y(1-x)\,dx \Rightarrow \frac{dy}{y} = \left(\frac{1-x}{x}\right)dx$

$\Rightarrow \frac{dy}{y} = \left(\frac{1}{x} - 1\right)dx$

By integrating, we have

$\int \frac{dy}{y} = \int \left(\frac{1}{x} - 1\right)dx \Rightarrow \log y = \int \frac{1}{x}dx - \int dx$

$\Rightarrow \log y = \log x - x + c \Rightarrow \log y - \log x = c - x$

$\Rightarrow \log \frac{y}{x} = c - x \Rightarrow \frac{y}{x} = e^{c-x}$

$\Rightarrow y = x e^{c-x}$ (Reqd. Soln)

Ex (3) Solve $\frac{dy}{dx} = e^{x-y} + x^2 e^{-y}$ [VVI]

Soln. Given eqn. can be $\frac{dy}{dx} = e^{x} \cdot e^{-y} + x^2 \cdot e^{-y}$

$\Rightarrow \frac{dy}{dx} = e^{-y}(e^{x} + x^2) \Rightarrow \frac{dy}{e^{-y}} = dx(e^{x} + x^2)$

$\Rightarrow e^{y}\,dy = e^{x}\,dx + x^2\,dx$, By integrating, we have

$\int e^{y}\,dy = \int e^{x}\,dx + \int x^2\,dx \Rightarrow e^{y} = e^{x} + \frac{x^3}{3} + c$

$\Rightarrow e^{x} + e^{y} + \frac{x^3}{3} = -c \Rightarrow e^{x} + e^{y} + \frac{x^3}{3} = K$ [$K = -c$]

Ex (4) Solve $\frac{dy}{dx} = \sqrt{1 + x^2 + y^2 + x^2y^2}$.

Soln. Given $\frac{dy}{dx} = \sqrt{1 + x^2 + y^2 + x^2y^2} = \sqrt{(1+x^2) + y^2(1+x^2)}$

$\Rightarrow \frac{dy}{dx} = \sqrt{(1+x^2)(1+y^2)} \Rightarrow \frac{dy}{\sqrt{1+y^2}} = (\sqrt{1+x^2})\,dx$

$\Rightarrow \int \frac{dy}{\sqrt{1+y^2}} = \int \sqrt{1+x^2}\,dx$

$\Rightarrow \log(y + \sqrt{1+y^2}) = \frac{x\sqrt{1+x^2}}{2} + \frac{1}{2}\log(x + \sqrt{1+x^2}) + K$

where K is constant